Enough and as Good: a Formal Model of Locke\textsuperscript{e}an First Appropriation

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Abstract

In developing a theory of the first appropriation of natural resources from the state of nature John Locke tells us that persons must leave “enough and as good” for others. Detailing exactly what this restriction requires divides right and left libertarians. Briefly, right libertarians interpret “enough and as good” as requiring no or very minimal restrictions on the first appropriation of natural resources, whereas left libertarians interpret “enough and as good” as requiring everyone be entitled to an equal share of unappropriated resources, able to claim no more beyond this equal share. This paper approaches the right versus left libertarian debate by developing a formal model that examines the welfare properties of different interpretations of the Locke\textsuperscript{e}an proviso. The model shows that underlying philosophical justifications for left libertarianism, when plausible assumptions hold, will actually be better served by a right libertarian proviso rather than a left libertarian one.
1 Introduction

John Locke begins chapter five of the *Second Treatise of Government* with a puzzle: God gave to mankind the whole earth in common, yet individuals own things. How does this happen? To which Locke responds: persons acquire property by mixing their labor with those unowned resources in the state of nature: “Whatsoever then he removes out of the state of nature that nature hath provided... he hath mixed his labour with, and joined to it something that is his own, and thereby makes it his property” (Locke 1690/1980: 19).

There are obvious objections to this theory of appropriation: what if someone tries to own *everything* by mixing her labor with the *entire* commons? Would it not be unfair for one person to own *so much*? To which Locke responds by setting provisos on his theory of appropriation. First, in the case of things capable of spoilage, persons may take “as much as anyone can make use of to any advantage of life before it spoils... whatever is beyond this, is more than his share, and belongs to others” (Locke 1690/1980: 20-21). Second, in the case of land and natural resources (things not subject to spoilage), persons may appropriate so long as they leave “enough and as good” for others (Locke 1690/1980: 21). Most think provisos of this general nature are essential for any theory of property. As Robert Nozick notes: “…any adequate theory of justice in acquisition will contain a proviso... A process normally giving rise to a permanent bequeathable property right in a previously unowned thing will not do so if the position of others no longer at liberty to use the thing is thereby worsened” (Nozick 1974: 178).

Though essential, what exactly these provisos permit and make impermissible is not obvious. Again following Nozick, clearly these provisos are “meant to ensure that the situation of others is not worsened” (Nozick 1974: 175). But as one of Nozick’s greatest detractors reminds: “Disagreement will come on what should here count as worsening another’s situation” (Cohen 1995: 75). This disagreement has led to an extensive literature debating the best way of understanding these provisos. Though they all take Locke as their root inspiration, these different versions of the provisos result in radically different conclusions concerning what is prohibited and what is permitted when it comes to the first appropriation of unowned resources from the state of nature.

This paper joins the debate by examining two different ways of interpreting the second Lockean proviso, the one pertaining to natural resources (land in particular), requiring we leave enough and as good for others. More specifically, the paper examines what we broadly call right libertarianism and left libertarianism. Briefly, right libertarians interpret “enough and as good” as requiring no or very minimal restrictions on the first appropriation of natural resources, and left libertarians interpret “enough and as good” as requiring everyone be
entitled to an equal share of unappropriated resources, able to claim no more beyond this equal share.

There are, of course, many ways of adjudicating between different interpretations of the Lockean proviso. First and foremost, one can analyze different versions of the proviso from an historical point of view: what exactly did Locke have in mind when he penned “enough and as good” for others? If one is less interested in the history of thought and more interested in developing a contemporary theory of property then one might evaluate different versions of the proviso from a moral point of view: which version seems most fair? Which version is favored by justice?

Our paper charts a third approach. Namely, we examine right and left versions of the Lockean proviso from an economic point of view. More specifically, we examine right and left libertarianism according to their welfare properties. We are thus mainly concerned with the consequences of implementing different restrictions on first appropriation. In *A Theory of Justice* John Rawls reminds us that “all ethical doctrines worth our attention take consequences into account in judging rightness. One which did not would simply be irrational, crazy” (Rawls 1971: 30). We agree, which is why we take the particular approach we do in this paper. Interestingly, though there has been significant effort invested into formally modeling Thomas Hobbes’s state of nature (Vanderschraaf 2006a; Vanderschraaf 2006b; Moehler 2009; Chung 2015), the authors cannot find one example of a formal model of Locke’s state of nature.¹ Hopefully this paper not only helps adjudicate between the right versus left libertarian debate, but also inspires further employment of the tools of economics and political science to examine more closely Locke’s state of nature and his theory of the social contract.

Using a standard general equilibrium framework taken from economic theory, we show that in the short term, a trade-off exists between leaving unowned land for future households that may be constrained in their ability to claim land in the first period, and improving the quality of land available for production in future periods. Therefore, if the value of investment in land is high or persistence in inequality of ability is high — thereby both raising the benefits and lowering the costs of allowing unconstrained appropriation — then the right libertarian proviso makes all households better off. We then proceed to show that in the long run, the right libertarian interpretation of the proviso is always Pareto dominant regardless the assumptions, so long as one adopts a long enough time horizon. Strikingly, this holds even in a world in which all land is claimed by one household in the first period, preventing households that would like to claim land in future periods from being able to do so, thereby making

¹One exception is a very recent working paper by Justin Bruner. However, Bruner formally models Locke’s account of the causes of conflict in the state of nature, not his theory of appropriation.
them initially worse off. Despite having fewer resources in the middle term, however, these households are eventually made better off by the efficiency gains (in terms of more efficient production) brought about by improvements in land via early investment. Generically, we show that no feasible compensation regime can save the left libertarian proviso from these long-run results.

Though we initially set out focusing on the welfare properties of competing theories of first appropriation, we end by examining how the results of our model bear on philosophical dimensions of the debate. We argue that our model raises problems for two — but certainly not the only possible — justifications for left libertarian schemes of property: luck egalitarianism and the Rawlsian difference principle. Insofar as one is either a luck egalitarian or Rawlsian, we believe our model shows that one ought not be a left libertarian.

2 Interpreting the Lockean Proviso

2.1 Right Libertarianism

What we consider to be right libertarianism can be broken down into three distinct subgroups. First (i) are those who are most radical, and simply deny that there is any proviso on the first appropriation of natural resources at all. Second (ii) are those who do believe that there should be some proviso restricting first appropriation, but interpret this proviso such that the restrictions are quite lax: here, few instances of first appropriation are impermissible. Third (iii) are those who agree with the second group that there is indeed an enough and as good proviso regulating first appropriation of unowned property, but interpret this proviso as requiring individuals appropriate resources from the commons. We include all three groups under the heading “right libertarianism” because we believe that implementing the proposals of all three groups leads to relatively similar states of affairs in terms of what, and how much, is ultimately appropriated. As such, the formal model we develop of the right libertarian theory of first appropriation likely captures — or captures closely enough — groups (i)-(iii), at least in terms of resulting consequences. Still, we acknowledge that there are important moral differences between groups (i)-(iii), and further remain agnostic as to which group (if any) best captures what Locke actually meant to say.

The first (i) group rejects the notion that there should be any proviso restricting first appropriation at all. There are various reasons why one might hold such a position. Murray Rothbard rejects any proviso because Rothbard agrees with Nozick that the proviso is best understood as not allowing appropriators to make the situation of others worse off, but “there is no way of measuring or knowing when [persons] are worse off or not” (Rothbard 1998: 244).
As another example, John T. Sanders argues that we should abandon the Lockeian proviso because it is self-defeating. On Sanders’s interpretation, the Lockeian theory of property is meant to make society more industrious, yet the proviso does just the opposite. In his words: “Abandoning the Lockeian Proviso altogether would have the effect of making more resources available, as potential property, to the class of initial labor mixers… Since the whole point of the Proviso was to promote opportunity for acquiring property, it seems to be self-defeating” (Sanders 1987: 382).

The second (ii) group contains those persons who do think first appropriation should be subject to a proviso, but think that this proviso imposes very limited restrictions. Nozick is often thought to be in this category, though we read Nozick’s discussion of the proviso to be rather speculative and non-committal. Following the literature, though, Michael Otsuka sets as his target what he calls “Nozick’s proviso,” defined as follows (Otsuka 2003: 23):

Nozick’s proviso. You may acquire previously unowned land (and its fruits) if and only if you make nobody else worse off than she would have been in a state of nature in which no land is privately held but each is free to gather and consume food and water from the land and make use of it.

As we mentioned in the introduction, much of the action in terms of debating different versions of the proviso is over how we ought to define exactly in what sense the proviso prevents us from making persons “worse off.” Yet Nozick is often interpreted as understanding “worse off” in a quite flatfooted way. For example, it is thought that on Nozick’s interpretation of the proviso, it is permissible for one individual to appropriate everything so long as that individual hires everyone else to work their newly acquired property, paying these persons a wage that is only slightly greater than the meager hand-to-mouth existence they would have led as hunter-gatherers existing in the commons (Cohen 1995: 79). This interpretation of Nozick serves as an example of what we consider a right libertarian proviso that does indeed impose some restrictions on first appropriation, though restrictions that are incredibly lax.

Finally, there is the third (iii) group. This group agrees that there is a proviso regulating the initial acquisition of property, but thinks that this proviso requires individuals appropriate from the state of nature. David Schmidtz is the leading thinker in this group: “far from permitting us to remove goods from the commons, the Proviso may sometimes require us to remove scarce goods from the commons” (Schmidtz 1980: 507). According to Schmidtz, the proviso to leave enough and as good for others requires individuals appropriate because those resources left in the commons will not – as most assume – remain in unused, pristine condition. Instead, a commons tragedy will result. With a system of property rights, though, property owners are incentivized to not overgraze their land, preserving resources
for future use. Because of this, "leaving resources in the commons does not leave enough and as good for others. The Lockean Proviso far from forbidding appropriation of resources from the commons actually requires appropriation under conditions of scarcity" (Schmidtz 1994/2008: 200). Again, we categorize those in groups (i)-(iii) as right libertarians because we believe implementation of their preferred interpretation (or lack thereof) of the Lockean proviso leads to relatively similar states of affairs: there will be much appropriation, subject to little, if any, restriction. This, we shall see, differs greatly from left libertarian interpretations.

2.2 Left Libertarianism

Like right libertarianism, left libertarianism is best understood as a cluster of views, all committed to, in some form, egalitarian ownership of natural resources as a starting baseline from which first appropriation then proceeds. Following Peter Vallentyne, Hillel Steiner, and Michael Otsuka, consider the following four ways of fleshing out left libertarian interpretations of the proviso on first appropriation:

(i) Natural resources might be owned in common in the sense that each person is free to use (but not appropriate) them as long as she is not violating the self-ownership rights of others. (ii) Natural resources might be jointly owned in the sense that any use, or perhaps only any appropriation, requires collected (e.g., majority) approval. (iii) Unilateral appropriation of unappropriated resources may be permitted as long as one pays to the members of society their per capita share of the full competitive value (based on supply and demand) of the resources that one claims. (iv) Unilateral appropriation of unappropriated resources may be permitted as long as one appropriates no more than is compatible with everyone having an equally valuable opportunity for a good life (Vallentyne, Steiner, and Otsuka 2005: 202-203).

What we consider under the label "left libertarianism" going forward will not include groups (i) and (ii), and may possibly exclude group (iv) as well (more on this below). Group (i) is excluded because it rejects permissible appropriation of any kind: though left libertarian interpretations of the Lockean proviso are more restrictive than right libertarian interpretations, most commonly endorsed theories of left libertarian appropriation allow appropriation

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2 Though we believe our model of right libertarianism captures Schmidtz's interpretation close enough, we do depart from Schmidtz in one key regard. Namely, in our model, the cost associated with unappropriated land is not that it is degraded, but rather that it is not being used productively. Modeling unappropriated land as worsening in quality would strengthen the benefits of the right libertarian proviso.
of some kind. We also exclude group (ii) because we believe the resulting state of affairs produced by implementation of the proviso proposed by group (ii) would look very different than those states of affairs produced by implementation of the provisos proposed by groups (iii) and (iv). Namely, we believe that the transaction costs of reaching agreement would be so high as to prevent much if not all appropriation from ever happening (Vallentyne and Vossen 2014). We are thus only concerned with left libertarian theories of first appropriation that do allow for appropriation (contra group (i)), that also allow for this appropriation to be unilateral (contra group (ii)). It should be noted, however, that most contemporary left libertarians are in groups (iii) and (iv), allowing the model we develop to still be quite general.

In group (iii) is the intellectual founder of contemporary left libertarianism, Steiner. According to Steiner:

Initially unowned things must be justly ownable. But how? The evident answer is that our equal original property rights entitle us to equal bundles of these things. That is, we each have a vested liberty to mix our self-owned labour with only as many of these things as would, in Locke’s famous phrase, leave “enough and as good” for others. And the correlative original duties vesting that liberty are ones not to appropriate more than this amount. We are each entitled to an equal share of (at least) raw natural resources. Mixing our labour with more than this share constitutes a relinquishment to our titles of that labour (Steiner 1994: 235-236).

A literal reading of this passage suggests that one may appropriate one’s equal share of natural resources and no more. According to Vallentyne, Steiner, and Otsuka, though, one may permissibly appropriate more than an equal share of unowned resources, so long as one compensates those whose equal share one has appropriated from. At first, our model does not address this possibility: each player is able to appropriate $1/n$ of the resources available, where $n$ is the total number of players in the appropriation game. This, we take it, is the left libertarian’s ideal state of affairs, and any post hoc redistribution that occurs when one takes more than one’s fair share is a second-best adjustment to non-ideal instances of first appropriation. We then extend the model, however, to include post hoc redistribution. As we shall see, adding such compensation to a left libertarian scheme of first appropriation does little to change the efficiency properties when compared to left libertarianism without post hoc compensation, which is a noteworthy result in and of itself.

Those in group (iv) do not endorse entitlement to an equal share of the world’s resources, but rather entitlement to a resource distribution ensuring everyone equal opportunity for liv-
ing a good life. Notable in this group is Otsuka, who introduces what he calls the “Egalitarian proviso” (Otsuka 2003: 24):

\[\textit{Egalitarian proviso.} \text{ You may acquire previously unowned worldly resources if and only if you leave enough so that everyone else can acquire an equally advantageous share of unowned worldly resources.}\]

Otsuka is non-committal concerning what is meant by “equally advantageous”: “The phrase ‘equally advantageous shares of unowned worldly resources’ that I employ in the egalitarian proviso should be read as a term of art that is a neutral among a range of familiar welfarist and resource-based metrics of equality” (Otsuka 2003: 25). Still, Otsuka does wish to make clear that an equally advantageous share of resources is not synonymous with an equal share of resources \textit{simpliciter}. In giving a thought experiment about appropriation of an unowned island, Otsuka rejects “the proposition that each person has an equal claim on the island’s resources. I would maintain that, \textit{ceteris paribus}, someone who would, through no fault of his own on account of his mental and physical constitution, be worse off in terms of welfare than another under an equal distribution of resources, has a greater claim on the island’s resources than another who would be better off than he in terms of welfare” (Otsuka 2003: 29).

Otsuka’s point is well-taken: if what we care about are equal levels of capability, then granting equal shares of natural resources is not sufficient to guarantee this. We are not sure how to formally model Otsuka’s interpretation of the Lockean proviso. This leaves two possibilities. First, one can interpret our model as including those in group (iii) and only group (iii). Or, one might grant that in the real world, when it comes time to actually implement restrictions on first appropriation or post hoc redistributions of what has already been appropriated, making nuanced welfare judgments of the kind Otsuka has in mind will be incredibly difficult, if not impossible. As such, some rough-and-ready proxy for welfare will need to be employed, and when this is done, a plausible (though not the only plausible) candidate is \textit{actual} shares of resources. As such, our formal model of left libertarianism is mainly meant to capture group (iii), and we leave it up to the reader to determine whether our model captures closely enough group (iv) for our conclusions to hold for Otsuka’s version of the proviso as well.
3 The Model

3.1 Households

There exists a fixed population of two households, \{A, B\}, in the state of nature, which for simplicity we assume live for a number of periods, \(T > 1\), indexed by \(t \geq 1\). Each household features a constant utility function in each period that is defined over their consumption of two goods \(x^i_t, y^i_t \geq 0\), their leisure \(l^i_t \in [0, \frac{1}{2}]\), and whether they invested in land this period \(I^i_t \in [0, 1]\), with a common discount rate \(\beta \in (0, 1)\). The only assumptions we make concerning utility functions – standard in the economic theory of general equilibrium – are that these functions are quasiconcave for each consumption good (meaning that there exist diminishing marginal returns for each good), that the two consumption goods are complements (meaning that the marginal return on one good is increasing in the amount one consumes of the other, so that one wants both of these goods), and that utility of leisure and not investing in land is linear and independent of consumption (meaning that consuming goods does not add to the utility of leisure, and that consuming goods does not take away from the disutility of labor).

Specifically, the utility function for each household can be written in the form:

\[
U^i_t(x^i_t, y^i_t, l^i_t, I^i_t) = u(x^i_t, y^i_t) + \omega_l t - c^i_t I^i_t
\]

where \(\omega > 0\) and \(c^i_t \geq 0\).

Households can work land to produce either of the two consumption goods. The production functions for each good (in terms of the land and labor required to produce this good) are as follows:

\[
\pi_x(l^i_t(x), L^i_t(x)) = \min\{l^i_t(x), L^i_t(x)\}
\]

(2)

\[
\pi_y(l^i_t(y), L^i_t(y)) = \gamma^i(\min\{l^i_t(x), L^i_t(x)\})
\]

(3)

where \(\gamma > 1\) and \(\tilde{t}\) is the number of periods in which the marginal unit of land has been invested. The more periods the land has been invested in the less labor it takes to produce good \(y\), though this is not so for good \(x\): the rate of production for \(x\) remains constant regardless how much the land has been invested in. We can think of \(x\) as a simple, natural consumption good (e.g., apples) that must only be harvested, while \(y\) is a good which is more amenable to structured production, and therefore mechanization (e.g., advanced

\(^3\)In the appendix, we show that all results extend to a setting with \(N > 2\) households.
agriculture). Thus, the more a parcel of land has been invested in—say, the more advanced one’s agricultural system is—the more of consumption good $y$ can be produced for the same amount of labor when compared to a parcel of land that has been invested in less. Intuitively, think here of how much labor it requires to produce a bushel of wheat on a primitive farm in a typical third world country when compared to the amount of labor it requires to produce that same bushel of wheat on a farm in Iowa.

The only difference between the two households concerns their ability to invest in land. In the first period, one household, $A$, will have cost function $c^1_A = 0$ and will therefore bear no cost from investing in land. The other household, $B$, has a cost of investment $c^1_B = \bar{c}$, where $\bar{c}$ is sufficiently large such that $B$ will never invest in the land.\(^4\) Thus, one household faces a low cost of investment, either because of easier resource accessibility, sheer ability, or other forms of luck and opportunity, and is thus the *advantaged type*. The other household faces an unbearably high cost of investment, and is thus the *disadvantaged type*. We make this assumption because, if there is no difference in ability to invest among households in the state of nature, then there will be no difference between the right and left interpretations of the Lockean proviso in terms of their efficiency properties. Assuming there are differences in ability to invest in land between the two households does not imply that these differences supervene on the innate ability or capacity of these households. The reason why $A$ has low investment costs in the first period might be because this household has physical access to fertile, green pastures, whereas the reason why $B$ has high investment costs in the first period might be because the land this household has physical access to is quite rocky and of general poor quality, making it difficult to invest in.

Though household $A$ begins as the advantaged type and household $B$ begins as the disadvantaged type these cost functions can vary over time. While one generation of a household may possess better opportunities or ability, there is no guarantee that the relative fortunes of the next generation will be the same. Given that a household $i$ is the advantaged type in period $t$, they will remain the advantaged type in period $t + 1$ with probability $p$. They will become the disadvantaged type (with household $j$ becoming the advantaged type) with corresponding probability $1 - p$. Therefore, we can think of $p$ as *persistence* in terms of place within the distribution of opportunities, and $1 - p$ as *mobility* in terms of place within the distribution of opportunities. The revelation of the next period’s skill distribution occurs just before consumption in the previous period.

Once the next period’s skill distribution has been revealed, the households, using wealth

\(^{4}\)An alternative interpretation of this assumption is that $B$ is a household which is not born until the second period, and hence cannot appropriate land in the first period, for $B$ simply has not been born yet. Therefore, the model as presented also embeds a model with population growth.
acquired from wages and land, can purchase either of the two goods and/or land for the next period. These will be sold at market prices in a standard general equilibrium framework, as neither party is a monopsony buyer nor a monopoly seller.

3.2 Land

In the state of nature there exists an (initially unowned) unit interval of land. In keeping with any (standard) interpretation of the Lockean proviso, the act of mixing one’s labor with some portion of land $L$ as investment confers a property right in that land. The household $i$ which appropriated the land will have control rights over that portion of land going forward. From that period onward, the household that owns the land will be able to invest in the land themselves, hire workers from the other household to invest in the land, decide what is done with any goods produced from the land, as well as have the ability to transfer control rights both as present-day rentals, and in future periods.

Of course, working the land for the purpose of further production inherently removes the ability of the general population to work that land and reap its returns. The key difference between competing interpretations of the Lockean proviso is precisely in how households can appropriate initially unowned land for their own purposes, as discussed above:

**Definition 1:** If property appropriation adheres to the right libertarian proviso (RLP), then any household $i$ can gain a property right in any unowned land.

**Definition 2:** If property appropriation adheres to the left libertarian proviso (LLP), then any household $i$ can only gain a property right by appropriation in $L \leq \frac{1}{2}$.

Both parties have full control rights over the goods that are produced from the land appropriated. There can be trade between the parties, which occurs at the culmination of each period. Control rights can also be transferred for future periods, such that land that was held by household $i$ in period $t$ will be owned by $j$ in period $t + 1$.

Neither household has any property rights at the beginning of the first period. This is important for, as the Coase theorem shows us, if there are no transaction costs then regardless the initial allocation of property rights, all inefficiencies will be bargained away and an efficient outcome reached (Coase 1960). Thus, if right and left libertarian interpretations of the proviso were about how property rights are initially allocated rather than about how one acquires property rights in the first place then there would be no difference between the two in terms of their efficiency properties (so long as we assume no transaction costs). But the debate is not about what the initial allocation of property rights should be, but rather about how we come to the initial allocation of property rights in the first place. As such, at the beginning of the first period there is nothing for the parties to bargain with.
To sum up, the timing of each period $t$ is as follows:

1. Any unowned land may be claimed (as allowed by the relevant version of the proviso) via investment, and any owned land may also feature further investment, either by the controlling household or through hired labor.

2. $\{c_{ti}^{t+1}\}_{i=A,B}$ is revealed.

3. Goods are produced using land and labor.

4. Trade of $x$, $y$, and $L_{t+1}$ occurs in a general equilibrium framework.

The equilibrium concept across periods is Markov Perfect Equilibrium (MPE), while the equilibrium concept within periods will be the standard Walrasian equilibrium. We will focus on strategies that map only from the state variables (the structure of land ownership, the level of previous investment $\int \gamma^i \partial L$, the ability of households to invest in the current period $\{c_t^i\}_{i=A,B}$, and the ability of the households to invest in the next period $\{c_{ti}^{t+1}\}_{i=A,B}$) onto strategies (investment decisions and general equilibrium production and trade).

4 Analysis

4.1 Preliminaries

An initial observation between the two differing versions of the Lockean proviso – which follows directly from definitions – is that more land will be claimed within the first period given the RLP when compared to the LLP. In particular, there is no reason for $A$ to leave any land unowned in the first period under the RLP, as it simply restricts their ability to sell the land for rents in the future.

Observation 1: All land will be claimed in period 1 by $A$ under the RLP.

$\frac{1}{2}$ of the land will be claimed by $A$ under the LLP.

This is a direct implication of $c_A^1 = 0$. Unconstrained, the advantaged household will claim the land which is too costly to acquire by the disadvantaged household. Hence, when the investment costs of the advantaged household are non-existent (which is by assumption true in the first period for $A$), they will claim everything that is left behind by the disadvantaged household facing high investment costs.

A corollary of this is that the RLP will maximize the amount of total investment over time. This will have a direct impact upon total production and thus total utility (that is, the sum of utility for both $A$ and $B$), as well as the utility of household $A$ in particular:
**Proposition 1:** The RLP, when compared to the LLP, will maximize total investment, total production, and the utility of A.

One argument made by economists for something resembling the RLP is that it is the *growth-maximizing* interpretation of the proviso. As investment increases, the labor required to produce a unit of good y goes down. Hence, even if labor remains fixed, total production still increases. This is not dissimilar to the standard argument for private property rights in general: by incentivizing investment, allowing private ownership of property will maximize the total production of utility in society.\(^5\)

Proposition 1 does not tell us, however, whether B is made better off given the increased investment under the RLP when compared to the LLP. In particular, since B is unable to claim land due to being the disadvantaged type in the first period, it is not *ex ante* obvious that B will be able to take advantage of this additional investment carried out by household A which occurs under the RLP. To actually consider the richer utility implications, one must analyze the equilibrium outcomes under both interpretations of the proviso. We begin by noting the relationship between investment in land, the labor wages in period \(t\) (\(w^t\)), and the prices of our two consumption goods in period \(t\) (\(p^t_x\) and \(p^t_y\)).

**Proposition 2:** The more investment in periods \(\{1, \ldots, t - 1\}\), the smaller \(w^t\), \(p^t_x\), and \(p^t_y\).

Proposition 2 says the more investment that has occurred the less labor will cost and the less our two consumption goods will cost. This arises due to investment reducing the price of consumable good y. As a result, the marginal cost of one unit of y falls, which lowers the wage necessary to hire workers, further reducing the price of both goods. Note that this does not mean that more investment intrinsically makes household B better off. If B owns no land, B will still get the same utility (due to the lower wage) even though goods x and y are cheaper.

To examine more carefully B’s welfare under the RLP compared to the LLP we need to understand how these wage and price dynamics highlighted in the previous result impact the allocation of land.

**Proposition 3:** Let \(c^{t+1}_i = 0\) and \(c^{t+1}_j = \bar{c}\). Then \(L^{t+1}_i \geq L^{t+1}_j\).

What Proposition 3 tells us is that the household that expects to gain more from investment in the next period (by being the advantaged type in the next period) will end up with a greater share of the total land than the other household in that period. This is because, while both households value the land equally for its rental value (i.e., the ability to earn

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\(^5\)For an overview see Acemoglu (2009).
profits by using the land to produce goods), the advantaged household in period \( t + 1 \) will be able to get an even larger market return on the land in period \( t + 1 \) due to increased productivity. While the less advantaged household could still make full use of the land by hiring the advantaged household to invest in it for them, they would need to pay them to do so, hence lowering their value of holding land.

4.2 Two Periods

We are now in a position to compare \( B \)'s welfare under the RLP and LLP. We do this first for a game lasting strictly two periods in the current subsection. The next subsection extends this analysis to an indefinite number of periods. In the two-period case, we can examine equilibrium outcomes by backwards induction. We do so by considering two possible cases: first, the case in which \( B \) remains the disadvantaged type in the second terminal period \( (c_B^2 = \tau) \). And second, the case in which \( B \) is now the advantaged type the second terminal period \( (c_B^2 = 0) \).

**CASE 1:** \( c_B^2 = \tau \).

Consider when \( B \) is again the disadvantaged type in the second period. Here, \( B \) will again be unable to invest in land. By Proposition 2, we know that the prices and wages will be lower under the RLP. However, since wages move stepwise with prices, these effects will net out and have no impact upon \( B \)'s total utility. Therefore, the real wage (in utility terms) for \( B \) will be the same.

The only change between the two periods comes in the form of available land and, particularly, the quality of this available land. By Proposition 1, the land available for \( B \) to purchase (should \( A \) be willing to sell any land) under the RLP at the end of first period will be of higher marginal value than the land which was available in the first period because it has been invested in. An alternative way to think about this is that the effective price of land will be lower under the RLP.

As a result of this, \( B \) can only be made better off under the RLP when compared to the LLP. If \( B \) buys no land under either regime because \( A \) is unwilling to sell, then \( B \) will have the same utility under both. However, if \( A \) is willing to sell and \( B \) buys land under the RLP (where the effective price is lower), \( B \) will get a greater return in terms of production capacity when compared to a world in which \( B \) buys land under the LLP. Households that stay perpetually disadvantaged can thus only be made better off by letting \( A \) claim (and invest in) all the land in the first period: for if \( B \) is able to buy land and \( A \) willing to sell, then the land \( B \) buys in the RLP world will be better then the land \( B \) buys in the LLP.
world. To conclude: \( B \) will always be weakly better off under the RLP when compared to the LLP when \( c_B^2 = \tau \).

**CASE 2:** \( c_B^2 = 0 \).

Now consider the case when \( B \) is the advantaged type in the second period. By Proposition 3, we know that \( B \) will own at least half of the land in the second, terminal period. Under the RLP all this land will be acquired by purchasing it from household \( A \) for, as Observation 1 notes, \( A \) claims all the land in the first period under the RLP. Under the LLP household \( B \) will still own at least half of the land in the second period. However, some (or, at the limiting case where \( B \) owns exactly half the total stock of land, all) of this land will be appropriated from the state of nature by \( B \) (as \( A \) could only take half the land in the first period) and thus will not cost \( B \) anything.

This freely available land might make \( B \) better off under the LLP when \( c_B^2 = 0 \). This does not immediately follow, however. For even though \( B \) gets free land under the LLP when \( c_B^2 = 0 \), this land has not been invested in. That is, the marginal value of this land will be less when compared to the land \( B \) buys from \( A \). Thus, whether \( B \) is better off under the LLP when \( c_B^2 = 0 \) depends on the effect investment has (how parameter \( \gamma \) is specified) on quality of land. If investment effects are sufficiently high then \( B \) will be better off under the RLP in this second case as well, for even though all the land \( B \) ends up owning is bought from \( A \), \( A \)'s investment in this land makes it valuable enough such that \( B \) is better off paying for this more productive land than being endowed with free, uninvested land.

There thus exists a general trade-off between the two regimes when examining strictly two periods. The RLP provides higher quality land, while the LLP provides equal access to land when there is mobility that will eventually allow \( B \) to invest in land in the second period as the advantaged type. The formal model allows us to make precise the circumstances under which each will maximize the welfare of \( B \):

**Theorem 1:** \( B \) will be made better off by the RLP when compared to the LLP with two periods if and only if:

- Investment quality \( \gamma \) is sufficiently high, or
- Persistence \( p \) is sufficiently high

Here is what we can conclude from Theorem 1. When investment quality \( \gamma \) is high, the benefit of greater investment in the first period is higher. Therefore, the RLP becomes a relatively more efficient regime regardless whether \( B \) is the advantaged type or disadvantaged
type in the second period. In addition, when persistence $p$ is high (i.e., mobility is low), the LLP's benefit (greater equality in ability to acquire land) will never obtain. Therefore, it is better for the perpetually disadvantaged type to simply allow $A$ to claim all the land in the first period and improve it: when households face larger and more permanent differences in natural ability, it is actually better to have a land appropriation regime which exploits these differences, rather than one that attempts to impose a form of equality that likely will never be helpful for either household, but particularly the disadvantaged household.

4.3 Long Run

We now examine how the welfare evaluations differ when extending the life of the households to several generations (i.e., $T > 2$). This is necessary when examining initial property acquisition, for we need to understand the long-run implications of initial acquisition over several generations, not just two time periods as the last section examined.

Begin by noting that until all land is owned, we know by the preceding section that everyone is made (weakly) better off under the RLP when compared to the LLP, as the additional land would never be used under the latter interpretation of the proviso. In addition, if in some period $\hat{t}$, $B$ becomes the advantaged type, in all future periods $\hat{t} + 1$ and forward, all land will be owned and traded under both the RLP and LLP. The only difference between the RLP and the LLP at period $\hat{t} + n$ will be the effective quality of the land. By Proposition 1, there will have been less investment under the LLP than the RLP. There will thus be higher-quality land under the RLP from this point forward. Since this reduces the price of labor and both our consumption goods, both households will be made better off from period $\hat{t} + 1$ forward under the RLP.

**Theorem 2:** There exists a $\hat{T} < \infty$ such that if $T > \hat{T}$, the RLP will make both households better off in $T$ when compared to the LLP. $\hat{T}$ is decreasing in all the same variables as in Theorem 1.

That is, given a long enough time horizon, even $B$ will be made better off given the RLP when compared to the LLP. So long as there are enough future periods to take advantage of the additional investment that occurs under the RLP, then even $B$ will see a rise in utility when compared to how $B$ fares under the LLP. The comparative statics are the same as with Theorem 1, as they are driven by the determinants of the price of land, and the benefits of additional investment.
4.4 Compensation

Thus far our interpretation of the left libertarian proviso says that households may claim $1/n$ of the available land, where $n$ is the number of players in the appropriation game. This, clearly, is the ideal articulated by what we called group (iii) in section 2.2 above. But we also noted in section 2.2 above that those in group (iii) allow for appropriators to take more than their $1/n$ share so long as they compensate others for doing so. We now enrich our model by redefining the left libertarian proviso to account for this, for it is intuitively plausible that permitting greater appropriation under the left libertarian proviso but then requiring transfers by those who took more than their fair share will allow the left libertarian proviso to better approximate the desirable welfare properties of the right libertarian proviso.

Definition 3: If property appropriation adheres to the alternative left libertarian proviso ($LLP^*$), then any household $i$ can gain a property right by appropriation in $L \leq \frac{1}{2}$ for free. For all land claimed above $\frac{1}{2}$, household $i$ must pay a flow transfer $\tau$ to the other household $j$.

Though it might prima facie seem that this will make a difference in terms of left libertarianism’s welfare properties, our model shows that this is not the case.

Corollary 1: There exists a $\hat{T} < \infty$ such that if $T > \hat{T}$, the RLP will make both actors sufficiently better off relative to $LLP^*$. $\hat{T}$ is decreasing in all the same variables as in Theorem 1.

The intuition behind Corollary 1 flows directly from our Theorem 2 above. Begin by noting that for any $\tau > 0$, $A$ will not invest in all of the land within the first period, as $A$ does in the RLP. As a result of this, given a long enough time horizon, the efficiency gains from early investment in land will eventually outweigh the temporary transfer gain in the first period by $B$ under the $LLP^*$, just as it outweighs $B$’s gain in the middle period under the standard interpretation of the $LLP$. The logic behind the result is thus exactly the same as Theorem 2: gains from early investment eventually pay off in a general equilibrium framework, which suggests that the earlier investment occurs, the better. Since forcing $A$ to pay a transfer to $B$ if $A$ takes more than $\frac{1}{2}$ of the available land under the $LLP^*$ has the effect of incentivizing $A$ to not claim all of the available land as $A$ does under the $RLP$, the result is the same.

4.5 Endogenous Persistence

The model thus far has bracketed away problems arising from persistent inequality; this is a function of our focusing exclusively on net welfare, not relative welfare. One potential
Concern with this approach lies in the possibility that household $i$ having greater relative wealth in period $t$ when compared to household $j$ will in turn increase the probability that household $i$ is the advantaged type in period $t + 1$ and household $j$ the disadvantaged type. In this way, we may worry that the increased inequality of the RLP will actually negate the growth benefits underpinning Theorem 2 if persistence depends upon holdings in the current period. The result, however, is robust to such an extension.

**Corollary 2:** Suppose $p_t \in (0, 1)$ is an increasing function of relative wealth in period $t$. There exists a $\hat{T} < \infty$ such that if $T > \hat{T}$, the RLP will make both households better off in $T$ when compared to the LLP. $\hat{T}$ is decreasing in all the same variables as in Theorem 1.

As long as persistence is bounded from above by 1 (i.e., it is at the very least possible for even a household that owns all the land to become disadvantaged) then, given a long enough time horizon, the initially disadvantaged household will eventually be better off under the RLP thanks to early investment effects. The result thus remains the same.

More interesting is the question of whether this endogenous persistence increases or decreases the advantages of the RLP. Surprisingly, there are cases where it actually increases the efficiency of the RLP:

**Proposition 4:** If $p_t$ is sufficiently concave, the $\hat{T}$ described in Corollary 2 will be smaller than the $\hat{T}$ in Theorem 2 when $E[p_t] = p_t$ from the standard model above.

What drives Proposition 4? Begin by recalling Case 1 above: if the initially disadvantaged household is perpetually disadvantaged then they are better off under the RLP, as they cannot appropriate regardless the regime they are in but at least have a lower effective price of land under the RLP. Therefore, if $p_t$ responds more to the first parcels of land acquired (i.e., is sufficiently concave) then persistence will be high under both the RLP and LLP, and Case 1 becomes more likely. Such being the case, the LLP leaves the disadvantaged household in the worst possible scenario: unlikely to ever become advantaged, but with relatively expensive access to land going forward (compared to the RLP). In this case, the RLP is even more beneficial for the worst-off when persistence is endogenous. If this condition does not hold (i.e., if $p_t$ is not sufficiently concave) then the extra inequality created by the RLP will further prevent the disadvantaged household from ever becoming advantaged, therefore extending the time horizon necessary for the RLP to become preferable when compared to Theorem 2. In either case, however, it is important to reiterate that the RLP will still eventually raise the welfare of the worst-off, as Corollary 2 shows.
4.6 Other Extensions

The model can be extended to include a more realistic examination of economies emerging out of the state of nature. Some extensions would improve the relative performance of the LLP, such as transaction costs for trade or monopolistic advantages in the market for land. However, as long as these do not become so large as to prevent all profitable trade, the main results will still hold and there will still exist a sufficiently long time horizon such that the RLP better maximizes the welfare of all parties when compared to the LLP.

In addition, many such extensions will actually make the wedge between the RLP and LLP greater. For example, adding interior costs of investment (i.e., making $\tau$ not rule out all investment and/or setting $c_1^A > 0$) limits the ability of $A$ to take everything in the first period. Similarly, greater degrees of specialization (i.e., in the ability to produce either of the two goods) or savings markets between the two periods would provide greater general equilibrium benefits to household $B$ from $A$’s initial investment. Our model actually understates the welfare benefits of the RLP on some dimensions.

5 The Foundations of Left Libertarianism

We now wish to further explore the philosophical implications of our model’s results by taking a closer look at the foundations of left libertarianism. What do we mean by “foundations”? Right and left libertarianism can be understood as different theories of property rights. But what justifies the employment of one specific set of property rights over another? What usually does the justificatory work here is an underlying normative theory or principle (Vossen 2009). One has an underlying normative principle $N$, and then argues that $N$ implies a specific way of organizing property rights. Maybe $N$ implies a left libertarian scheme of property rights; perhaps a different normative principle $N^*$ implies a right libertarian scheme of property rights.

As we noted in section 2.2, left libertarianism is a cluster of views all broadly committed to a relatively strict interpretation of “enough and as good” – that is, left libertarianism can be understood as a set of related ways of organizing property rights. On its own, the term “left libertarianism” implies no underlying normative commitments. Just as there are many different ways of specifying the left libertarian theory of first appropriation, there are also many different underlying normative principles that are used to justify a left libertarian scheme of property rights. One theorist might argue for left libertarianism with normative principle $N$, another theorist might also argue for left libertarianism with distinct normative principle $N^*$. In what follows we want to discuss two underlying justifications for left libertari-
ianism. We argue that our model causes problems for such justifications. This should not be interpreted as our model causing problems for left libertarianism generally speaking. Insofar as one justifies a left libertarian scheme of property rights with an underlying normative principle that we do not discuss, there are no immediate conclusions to draw concerning our model and left libertarianism.

Many theorists justify a left libertarian scheme of property rights via some form of luck egalitarianism (e.g., Vallentyne 2002; Otsuka 2003: 23-27; Steiner 2011: 110). Roughly, luck egalitarianism is a normative theory saying that “an unequal distribution whose inequality cannot be vindicated by some choice or fault or desert on the part of (some of) the relevant affected agents is unfair, and therefore, pro tanto unjust” (Cohen 2008: 7). That is, luck egalitarianism holds that deviations from perfect equality are only justified if the deviations are the result of non-arbitrary factors about the affected parties. Many luck egalitarians endorse some scheme of left libertarian property rights precisely because they believe such a scheme of property rights mitigates unequal distributions arising from undeserved differences: it prevents people, for example, from owning more than others simply because they were the advantaged type in the first period. Thus, we have a conditional statement: if one is a luck egalitarian then one should be a left libertarian.

Our model indicts such justifications of left libertarianism. For it shows that there will be inequality given a left libertarian interpretation of the Lockean proviso solely due to differences in the ability to invest in land. In our model the two households do not acquire equal distributions given the left libertarian proviso in either the two-stage analysis or the long-run analysis. This difference in what they acquire is solely grounded in whether they are the advantaged type or disadvantaged type in the first period which, we think, is an arbitrary feature of the two households. So left libertarianism (under plausible assumptions about differences in ability) will not eliminate inequalities grounded in brute luck.

Now here it might be argued that, though our model shows that both right and left libertarian interpretations of the Lockean proviso result in arbitrary inequalities, our model also shows that left libertarianism results in less inequality when compared to right libertarianism. Thus, left libertarianism can be seen as something of a second-best institutional arrangement for luck egalitarianism: though it does not eliminate arbitrary inequalities it can minimize them, at least when compared to right libertarianism.

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More specifically, here are plausible two justifications for left libertarianism we do not address. First, one might think that it is normatively important for all persons to in a sense be equal stakeholders of the world: we should all get the chance to hold some property. As our model shows, under right libertarianism there will be some set of parameters such that the initially disadvantaged never gets to own property. And second, one might endorse some kind of relational egalitarianism of the kind proposed by Elizabeth Anderson (1999), where the goal is to eliminate forms of inequality that allow for domination. We are unsure of what (if anything) our model has to say about this justification of left libertarianism.
This is not necessarily true. As we saw in the two-stage analysis in §4.2 (Case 1), if there is high persistence in differences in ability then there may be less inequality under right libertarianism. This is because the advantaged type creates more net resources and provides more high-quality land for the disadvantaged type to make use of. So left libertarianism not only does not eliminate inequalities grounded in brute luck, but it also does not necessarily reduce unjustified inequalities either. Thus, the conditional statement does not hold. If one is a luck egalitarian one should not necessarily be a left libertarian; indeed, depending on expected rates of persistence, a case can be made that one ought to be a right libertarian instead.

But suppose for the sake of argument that left libertarianism always minimized inequalities arising from brute luck when compared to right libertarianism. Still, our model creates an interesting dilemma for those luck egalitarians who wish to embrace left libertarianism under such a presumption. The leveling down objection to egalitarianism confronts the egalitarian with two states of affairs: the first (a) where there is perfect equality, and the second (b) where there is inequality, yet such inequality Pareto dominates the first, completely equal state of affairs. If one is committed to equality as the luck egalitarian is then one should endorse state of affairs (a). Yet, intuitively, this does not seem right. It seems here that one should endorse state of affairs (b) as better than state of affairs (a): after all, how could (b) be worse than (a) when everyone is better off in (b) than they are when compared to (a)?

If the luck egalitarian endorses left libertarianism because it minimizes inequalities that arise from brute luck when compared to right libertarianism then our model shows that they must essentially endorse state of affairs (a) over state of affairs (b). For, by Theorem 2, right libertarianism Pareto dominates left libertarianism regardless investment quality and mobility if one adopts a long enough time horizon. Now clearly there will be some who are willing to do this – who are willing to say that equality of outcome in terms of minimizing differences resulting from brute luck cannot be outweighed by superior welfare considerations.⁷ There is nothing wrong with harboring this intuition. Many will have the opposite intuition, however. For many, accepting state of affairs (a) as better than state of affairs (b) is simply too big a bullet to bite. If one joins this crowd and refuses to bite such a bullet, then our model shows that one cannot endorse left libertarianism via some kind of luck egalitarian argument.

In response to the problems associated with adopting luck egalitarian foundations, one move the left libertarian can make here is to adopt different theoretical foundations. One recent attempt to do this has been pursued by Jonathan Quong (2011). On Quong’s view,⁷

⁷Returning to the footnote immediately above, the relational egalitarian could plausible argue for (a) over (b) if the inequality in (b) was great enough to allow for relations of domination.
instead of adopting some form of luck egalitarianism, left libertarians ought to endorse a Rawlsian-inspired theory of justice as reciprocity; justice as reciprocity, according to Quong, leads one to left libertarianism. The idea behind justice as reciprocity is to ensure a fair distribution of the burdens and benefits of social cooperation. Famously, Rawls thought justice as reciprocity led to the difference principle, which holds that the basic structure of society must be organized so that it maximizes the welfare of those least advantaged in society. But, as Quong notes, there are other distributive principles one could adopt besides the difference principle from the perspective of justice as reciprocity. We consider both cases: where justice as reciprocity leads to the difference principle, and where justice as reciprocity leads to some distributive principle other than the difference principle.

Consider the first case, where justice as reciprocity is fleshed out with the difference principle. By Theorem 2, right libertarianism Pareto dominates left libertarianism regardless of investment quality and mobility in the long run. Thus, if one embraces the difference principle, and if one adopts a long-run view of welfare over generations, then one must be a right libertarian rather than a left libertarian, for the right libertarian proviso will make the least disadvantaged better off when compared to the left libertarian proviso. Hence, if one fleshes out the idea of justice as reciprocity with the difference principle, then justice as reciprocity does not lead one to left libertarianism: it does not, in fact, maximize the welfare of the least advantaged.

But what if justice as reciprocity is fleshed out with a distributive principle besides the difference principle? Since the idea of justice as reciprocity includes the idea of mutual advantage (Rawls 1993/2005: 16-17), any plausible interpretation of justice as reciprocity will require, at the very least, a Pareto efficient distribution: if all parties could be made better off, the idea of mutual advantage requires that such gains are exhausted. Indeed, Rawls's first articulation of the idea of justice as reciprocity did not require the difference principle specifically (the difference principle represents one point on the Pareto frontier), but permitted any form of inequality so long as it Pareto dominated equality (Rawls 1958/1999: 50). But note, our model shows that right libertarianism does Pareto dominate left libertarianism if one adopts a long enough time horizon. Thus, so long as justice as reciprocity includes the idea of mutual advantage, and so long as mutual advantage requires we take Pareto gains when they are available, then even rejecting the difference principle as the most plausible interpretation of justice as reciprocity fails to save the left libertarian.

Note though that we cannot necessarily conclude that the right libertarian proviso is the uniquely best scheme of institutions from the perspective of justice as reciprocity. Any claim to Pareto optimality is always contingent on the alternatives one is comparing the putative optimal state to. If the relevant comparisons were just right and left libertarianism
as defined in our model then the right libertarian proviso would be the uniquely best option according to any form of justice as reciprocity. But there could be a third, mixed scheme of property institutions we have not yet considered that Pareto dominates right libertarianism as we have defined it. Still, we can conclude that justice as reciprocity does not entail the left libertarian proviso, so long as right libertarianism as defined in our model is one of the eligible schemes of institutions to implement.

References


